THURSDAY, DECEMBER 14, 2017 AP CHEMISTRY

CHAPTER 21 SUPPLEMENTAL NOTES

- WHAT IS A LOGARITHM?

IT IS A WAY OF ASKING, WHAT POWER OF A CERTAIN NUMBER (CALLED THE BASE OF THE LOGARITHM) GIVES THE NUMBER "INSIDE" THE LOGARITHM?

$\log_{10} 1 = 0$ $\quad \log_2 1 = 0$ $\quad \text{power}_7\, 7 = 1$

$\log_{10} 10 = 1$ $\quad \log_2 2 = 1$ $\quad \text{power}_7\, 49 = 2$

$\log_{10} 100 = 2$ $\quad \log_2 4 = 2$ $\quad \text{power}_7\, 343 = 3$ ETC

WHAT ARE THEY FOR?

LOGARITHMS ARE DESIGNED TO TAKE A HARD PROBLEM (MULTIPLYING LARGE NUMBERS) AND MAKE IT AN EASY ONE (ADDING SMALL NUMBERS).

① $10^7 \times 10^5 = ?$ ② $\log_{10}(10^7 \times 10^5) = \log_{10} 10^5 + \log_{10} 10^7$

③ $5 + 7 = 12$ ④ 10^{12} (OR, ANTILOG 12)

$57,000,000,000 \times 42,000,000$

$\log_{10} 57,000,000,000 + \log_{10} 42,000,000$

$10.7558 + 7.6232 = 18.379$

$10^{18.379} = 2,390,000,000,000,000,000$

THINK OF LOGARITHMS AS THE REVERSE OF EXPONENTIATION

$a + b = c$ REVERSE: $a = c - b$

$a \cdot b = c$ REVERSE: $a = c \div b$

$a^b = c$ REVERSE: $\log_a a^b = b = \log_a c$

PROPERTIES OF LOGARITHMS

$$\log(M \times N) = \log M + \log N \quad (a^b \cdot a^c = a^{b+c})$$

$$\log(M \div N) = \log M - \log N \quad \left(\frac{a^b}{a^c} = a^{b-c}\right)$$

$$\log(M^x) = x \log M \quad ((a^b)^c = a^{b \cdot c})$$

$$a^{\wedge \log_a M} = M$$

ON YOUR CALCULAITO YOU HAVE TWO LOGARITHM BUTTONS.

1. log OR LOG MEANS $\log_{10}$

   (ANTILOG LOOKS LIKE 10^x)

2. ln MEANS $\log_e$ WHERE e IS A MATHEMATICAL CONSTANT $e \cong 2.71828\ldots$

FOR EXPONENTIAL DECAY, THE ALGEBRAIC EXPRESSION IS GIVEN IN TERMS OF e: $y = e^{-x}$

THIS IS WHY WE USE ln, ALSO CALLED THE NATURAL LOGARITHM: $\ln y = -x$

FOR RADIOACTIVE DECAY PROBLEMS WE USE ln.

$$\frac{N}{N_0} = e^{-kt} \qquad \ln(N/N_0) = -kt$$

Chapter 21 Supplemental Notes, continued.

Rates of Decay — Deriving the equation

In nuclear decay a measured rate (as in, decays per second) is proportional to the number of atoms (N).

$$\text{RATE} = -kN$$

or

$$\frac{\Delta N}{\Delta t} = -kN$$

or, in the language of calculus

$$\frac{dN}{dt} = -kN$$

Since the atoms are reducing their numbers over time, the expression includes a negative sign.

The equation $\frac{dN}{dt} = -kN$ tells use how rate changes as a function of the number of atoms.

We want to know how the number of atoms changes as a function of time. Integral calculus provides that function.

1. $\frac{dN}{dt} = -kN$

2. $\frac{1}{N}dN = -k\,dt$

3. Definite integral

$$\int_{N_0}^{N} \frac{1}{N}\,dN = -\int_{0}^{t} k\,dt$$

4. $\ln(N) - \ln(N_0) = -kt$

5. $\ln\left(N/N_0\right) = -kt$

6. $\boxed{\frac{N}{N_0} = e^{-kt}}$

N/N_0 is the decimal fraction of atoms that remain at time t.